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A nonsupersymmetric matrix orbifold

Tom Banks

Department of Physics and Astronomy
Rutgers University, Piscataway, NJ 08855-0849
E-mail: banks@physics.rutgers.edu

Luboš Motl

Department of Physics and Astronomy
Rutgers University, Piscataway, NJ 08855-0849
E-mail: motl@physics.rutgers.edu

Abstract: We construct the matrix description for a twisted version of the IIA string theory on S^1 with fermions antiperiodic around a spatial circle. The result is a 2 + 1-dimensional $\mathrm{U}(N) \times \mathrm{U}(N)$ nonsupersymmetric Yang-Mills theory with fermionic matter transforming in the $(\mathbf{N}, \bar{\mathbf{N}})$. The the two $\mathrm{U}(N)$'s are exchanged if one goes around a twisted circle of the worldvolume. Relations with type-0 theories are explored and we find type-0 matrix string limits of our gauge theory. We argue however that most of these results are falsified by the absence of SUSY nonrenormalization theorems and that the models do not in fact have a sensible Lorentz invariant space time interpretation.

Keywords: M-Theory, String Duality, Superstring Vacua, M(atrix) Theories.

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Contents

1. The conformal field theory description

Matrix Theory [1] has been used to describe M-theory with 32 supercharges in 8,9,10 and 11 dimensions as well as various projections of this theory. In this paper we would like to study a nonsupersymmetric Matrix model in order to obtain a better understanding of SUSY breaking in string theory. The problems of the model that we study show that SUSY breaking leads to rather disastrous consequences. However, we point out in the conclusions that the restriction to the light cone frame prevents us from abstracting completely clearcut lessons from this exercise.

Before proceeding, we note that after this paper was completed (but before we had become convinced that the results were worth publishing) another paper on Matrix models of nonsupersymmetric string theories appeared [18]. We do not understand the connection between the model presented there and the one we study. Another recent paper on nonsupersymmetric compactifications, with considerations related to ours is [19]. Our results do not agree with the suggestion of these authors that nonsupersymmetric compactifications lead to Poincaré invariant physics.

Let us start with the description of the conformal field theory that describes the model we are interested in. We start with the type-IIA theory. Compactification on a circle of radius R can be described as modding out the original theory by a symmetry isomorphic to $\mathbb{Z}$, consisting of the displacements by $2\pi kR$, $k \in \mathbb{Z}$, in the chosen direction. We can write those displacement as $\exp(2\pi ikR\hat{p})$. A "GSO-like" projection by this operator now guarantees that the total momentum of a string is a multiple of $1/R$. We also have to add "twisted sectors" where the trip around the closed string is physically equivalent to any element of the group that we divided by. Those sectors are wound strings, $X(\sigma + 2\pi) = X(\sigma) + 2\pi Rw$, where $w \in \mathbb{Z}$ is the winding number.

Such a compactification preserves all 32 supercharges. We will study a more complicated model which breaks the supersymmetry completely. The symmetry isomorphic to $\mathbb{Z}$ will be generated by (the direction of the circle is denoted by the index 2)

$$G = \exp(2\pi iR\hat{p}_2)(-1)^F , \tag{1.1}$$

where $(-1)^F$ counts the spacetime statistics (or spin; in the Green-Schwarz formalism it is also equivalent to the worldsheet spin). Because of this extra factor of $(-1)^F$ the physics becomes very different. The fermionic fields of the spacetime effective field theory become antiperiodic with the period $2\pi R_2$ while bosons are still periodic. Such a boundary condition of course breaks supersymmetry completely because it is impossible to define the (sign of the) supercharge everywhere. Those antiperiodic boundary conditions for the fermions are the same as those used in finite temperature calculations, with Euclidean time replaced by a spatial circle. This compactification, introduced in [7] is motivated by Scherk-Schwarz compactifications of supergravity [6]. The physical spectrum is obtained by requiring $G\,|\psi\rangle = |\psi\rangle$ and in the twisted sectors corresponding to G^w, $w \in \mathbb{Z}$, a trip around the closed string is equivalent the shift by $2\pi wR_2$ *times* $(-1)^{wF}$. Because $(-1)^{wF}$ for odd w anticommutes with fermions in the Green-Schwarz formalism, the fermions θ must be antiperiodic in the sectors with odd winding number. Similarly, the GSO projection $G\,|\psi\rangle = |\psi\rangle$ now does not imply that p_2 must be a multiple of $1/R_2$. Looking at (1.1) we see that there are two possibilities. Either $(-1)^F$ is equal to $+1$ and $p_2 = n/R_2$ *or* $(-1)^F = -1$ and $p_2 = (n + 1/2)/R_2$. We have sectors with p_2R_2 both integer or half-integer, but for integer p_2R_2 we project out all the fermions and for half-integer p_2R_2 we project out all the bosonic states.

Thus we have four kinds of sectors; odd or even w can be combined with integer or half-integer p_2R_2. For even w the boundary conditions are as in the untwisted theory but for odd values of w we must impose *antiperiodic* boundary conditions for the Green-Schwarz fermions θ.

Since the sectors with even values of w are well-known (we just keep only bosons or only fermions according to p_2), we note only that in the sectors with odd values of w the ground state has $8 \times (-1/24 - 1/48) = -1/2$ excitations both in the left-moving and right-moving sector (the same as the ground state of a NS-NS sector in the RNS formalism). In other words, the ground state is a nondegenerate bosonic tachyon. We must also take into the account the condition "$L_0 = \tilde{L}_0$", more precisely ($N_L = N_R = 0$ for the ground state of even w and $N_L = N_R = -1/2$ for odd w and we define $n = p_2 R_2$)

$$N_L = N_R + nw \,. \tag{1.2}$$

Furthermore for integer n we must project all the fermions out of the spectrum. For even w (which means also even nw) this leaves us with the bosonic states of the untwisted IIA theory. For odd w (which implies integer nw) the fermionic modes are half-integers and we see that due to (1.2) the number of left-moving and right-moving fermionic excitations must be equal $\mod 2$. Therefore the level matching condition (1.2) automatically projects out all the fermionic states.

Similarly for half-integer n we must get rid of all the bosonic states. If w is even, nw is integer and odd and apart from (1.2) we must also independently impose the condition $(-1)^F = -1$ and we get just the fermionic part of the spectrum of the type-IIA string theory. However for w odd (which implies that nw is half-integer and the fermions are antiperiodic), we see a mismatch $1/2$ modulo 1 between N_L and N_R in (1.2), so the bosons are projected out automatically as a result of the level-matching condition (1.2).

In both cases we saw that the $(-1)^F$ projection was automatic in sectors with odd values of w. It is a general property of orbifolds that in the twisted sectors where a trip around the closed string is equivalent to the symmetry g, the GSO projection $g\,|\psi\rangle = |\psi\rangle$ is a direct consequence of the level-matching condition.

What about the tachyons? For even values of w we have a part of the spectrum of type-IIA strings, so there is no tachyon. However for odd values of w, we can find a tachyon. Recall that

$$m^2 = \frac{4}{\alpha'} N_L + \left(\frac{n}{R_2} - \frac{wR_2}{\alpha'}\right)^2 = \frac{4}{\alpha'} N_R + \left(\frac{n}{R_2} + \frac{wR_2}{\alpha'}\right)^2 . \tag{1.3}$$

For $n = 0$, $w = \pm 1$, we see that the ground level $N_L = N_R = -1/2$ is really tachyonic for $R_2^2/\alpha' < 2$. For sufficiently small radius R_2 there is a bosonic tachyon in the spectrum. If R_2 is really small, also states with $\pm w = 3, 5, \ldots$ (but always $n = 0$) may become tachyons.

However for $n = 1/2$ (fermionic sector) and $w = 1$ we see from (1.2) that the lowest possible state has $N_R = -1/2$ and $N_L = 0$ (one $\theta_{-1/2}$ left-moving excitation) which means that m^2 expressed in (1.3) is never negative. This means that the tachyons can appear only in the bosonic spectrum (as scalars).

There are many interesting relations of such a nonsupersymmetric theory with other theories of this kind. For example, by a Wick rotation we can turn the twisted spatial circle into a time circle. The antiperiodic boundary conditions for the fermionic field then describe a path integral at finite temperature. The appearance of the tachyon in the spectrum for $R_2 < \sqrt{2\alpha'}$ is related to the Hagedorn phase transition. The infinite temperature, or zero R_2 limit of our model gives the type-0 theories.

The type-0 theories (-0A and -0B) are modifications of the type-II theories containing bosonic states only in "diagonal sectors" NS-NS and RR; we have also only one GSO projection counting the number of left-moving minus right-moving fermionic excitations. The type-II theories can be obtained as an $\mathbb{Z}_2$ orbifold (making separate projections on left-moving fermions) of the type-0 theories in the R-NS formalism; the difference between type-IIA and type-IIB theories is a sign of the projection in the RR sector; type-IIA is in a sense type-IIB with a discrete torsion. Equivalently, we can also obtain type-0 theories by orbifolding type-II theories but in the Green-Schwarz formalism: type-0A and -0B theories can be described in the Green-Schwarz formalism by the same degrees of freedom as the corresponding type-II theories, but we must include both PP and AA sector and perform the corresponding (diagonal) GSO-projection.

2. The matrix model

In a first naive attempt to construct a model describing the type-0A theory, we would probably make a local orbifold (orbifolding in Matrix theory was described in [9]) of the original Matrix Theory, corresponding to the $\mathbb{Z}_2$ orbifold of the worldsheet theory in the Green-Schwarz formalism. We would represent the operator $(-1)^F$ by a gauge transformation e.g. $\sigma_3 \otimes \mathbb{1}$ and the bosonic matrices would then be restricted to the block diagonal form, reducing the original group $\mathrm{U}(2N)$ into $\mathrm{U}(N) \times \mathrm{U}(N)$ with fermions in the off-diagonal blocks i.e. transforming as $(\mathbf{N}, \bar{\mathbf{N}})$. However the coordinates X of the two blocks would suffer from an instability forcing the eigenvalues of the two blocks to escape from each other: the negative ground-state energy of the "off-diagonal" fermions is not cancelled by a contribution of bosons and therefore the energy is unbounded from below even for finite N.

Bergman and Gaberdiel however pointed out [13] that it is more appropriate to think about type-0A string theory as a type-IIA theory orbifolded by a $\mathbb{Z}_2$ group generated by the usual $(-1)^F$ *times* the displacement by half of the circumference of the corresponding M-theoretical circle. Of course, this displacement could not be seen perturbatively. We are clearly led to the Scherk-Schwarz compactification of M-theory. We will thus attempt to construct a more sophisticated matrix model. We will find a model which naively incorporates all of the duality conjectures of Bergman and Gaberdiel and has type-0A,B and Rohm compactified type-IIA,B matrix string limits. In the end, we will find that many of our naive arguments are false, due to the absence of SUSY nonrenormalization theorems, and that the model we construct

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does not have a Lorentz invariant large-N limit. We argue that this implies that Scherk-Schwarz compactified M-theory does not have a Lorentz invariant vacuum.

Let us start with a review of untwisted M-theory compactified on a circle. The algorithm [9] to mod out the BFSS model by a group of physical symmetries H is to enlarge the gauge group $\mathrm{U}(N)$ and identify elements of H with some elements g of the gauge group. It means that the matrices $Y = X, \Pi, \theta$ are constrained to satisfy

$$h(Y) = g_h Y g_h^{-1}\,, \qquad g_h \in \mathrm{U}(N)\,, \quad h \in H\,. \tag{2.1}$$

We wrote gYg^{-1} because Y transform in the adjoint representation and the physical action of the symmetries on Y is denoted $h(Y)$. To obtain the $O(N)$ matrix model describing a single Hořava-Witten domain wall, we can set $g_h = \mathbb{1}$ and just postulate Y to be symmetric with respect to the symmetry (consisting of the reflection of X^1, Π^1, multiplying spinors θ by γ_1 and transposing all the matrices). Therefore X^1 and half θ's become antisymmetric hermitean matrices in the adjoint of $O(N)$ while the other X's and θ's become symmetric real matrices. The naive matrix description of the heterotic strings on tori together with the sectors and GSO-like projections on the heterotic matrix strings was obtained in [11].

Compactification of X^2 on a circle with radius R_2 can be done in a similar way. We just postulate the set of possible values of the $\mathrm{U}(N)$ indices to be $\{1, 2, \ldots, N\} \times (0, 2\pi)_{\text{circle}}$ and represent the physical symmetry $\exp(2\pi i R_2 \hat{p})$ by the gauge transformation $\mathbb{1} \otimes \exp(i\sigma_2)$. Note that the matrices now have two discrete and two continuous "indices". Postulating (2.1) tells us that the matrices must commute with any function of σ:

$$X^2_{mn}(\sigma_2, \sigma_2') = X^2_{mn}(\sigma_2)\delta(\sigma_2 - \sigma_2') - i\delta'(\sigma_2 - \sigma_2') \tag{2.2}$$

and similarly for the other matrices X^i and θ (without the $\delta'(\sigma_2 - \sigma_2')$ term). Now if we understand the summation over the sigma index as integration and ignore the factor $\delta(0)$ in the trace, the BFSS hamiltonian becomes precisely the hamiltonian of SYM theory with σ_2 being an extra coordinate.

"Matrices" of the form (2.2) can be also expressed in the terms of the Fourier modes as done first by Taylor [3]. The extra Fourier mode indices replacing σ_2, σ_2' are denoted M, N and (2.2) becomes

$$(X^2_{M+1,N+1})_{mn} = (X^2_{M,N})_{mn} + 2\pi R_2 \delta_{M,N}\delta_{mn} \tag{2.3}$$

and similarly for the other matrices without the last term.

We will study a compactification of M-theory on a twisted T^2 so we will use two worldvolume coordinates σ_1, σ_2 to represent those two circles. What about the $(-1)^F$ twist which modifies the compactification of X^2? Shifting both ends of the open strings $M, N \to M+1, N+1$ as in (2.3) must be accompanied by $(-1)^F$ which commutes with bosons but anticommutes with spacetime fermions. In the Green-Schwarz formalism it also anticommutes with the θ's. So the structure of the bosonic matrices is unchanged and the condition for θ's will be twisted:

$$(\theta_{M+1,N+1})_{mn} = -(\theta_{M,N})_{mn}. \tag{2.4}$$

In the continuous basis this is translated to

$$\theta_{mn}(\sigma_2, \sigma_2') = \theta_{mn}(\sigma_2)\delta(\sigma_2 - \sigma_2' + \pi). \tag{2.5}$$

This can be described by saying that θ has nonzero matrix elements between opposite points of the σ_2 circle. We will often use this "nonlocal" interpretation of the resulting theory even though the theory can be formulated as a conventional nonsupersymmetric gauge theory with fermionic matter, as we will show in a moment.

2.1 The $\mathrm{U}(N) \times \mathrm{U}(N)$ formalism

In order to get rid of the nonlocality, we must note that if we identify the opposite points with σ_2 and $\sigma_2+\pi$, so that σ_2 lives on a circle of radius π, everything becomes local. By halving the circle, we double the set of bosonic fields. The two $\mathrm{U}(N)$ groups at points σ_2 and $\sigma_2+\pi$ are completely independent, so that the gauge group becomes $\mathrm{U}(N) \times \mathrm{U}(N)$. We should also note that if we change σ_2 by π, the two factors $\mathrm{U}(N)$ exchange; this is an important boundary condition.

The bosonic fields thus transform in the adjoint representation of $\mathrm{U}(N) \times \mathrm{U}(N)$. What about the fermions θ? We saw that the two "matrix indices" σ_2 and σ_2' differ by π. One of them is thus associated with the gauge group $\mathrm{U}(N)$ at point σ, the other with the $\mathrm{U}(N)$ at point $\sigma+\pi$ which is the other $\mathrm{U}(N)$ factor in the $\mathrm{U}(N) \times \mathrm{U}(N)$ formulation. In other words, θ's transform as $(\mathbf{N}, \bar{\mathbf{N}})$ under the $\mathrm{U}(N) \times \mathrm{U}(N)$. This is a complex representation of (complex) dimension N^2 and the complex conjugate $\theta^\dagger$s transform as $(\bar{\mathbf{N}}, \mathbf{N})$. In the old language, θ and $\theta^\dagger$ differed by π in σ_2 or in other words, they corresponded to the opposite orientations of the arrow between σ_2 and $\sigma_2+\pi$. The number of real components is $2N^2$ (times the dimension of the spinor 16), the same as the dimension of the adjoint representation. This should not surprise us since for $R_2 \to \infty$ we expect that our nonsupersymmetric model mimics the physics of the supersymmetric model.

We could also check that the commutation relations derived from the "nonlocal" orbifold formulation agree with the canonical commutation relations of the $\mathrm{U}(N) \times \mathrm{U}(N)$ nonsupersymmetric gauge theory with the matter in $(\mathbf{N}, \bar{\mathbf{N}})$. These theories are very similar to the "quiver" theories of Moore and Douglas [10], but with a peculiar boundary condition that exchanges the two $\mathrm{U}(N)$ groups as we go around the twisted circle.

2.2 Actions for the local and nonlocal formulations

We will be considering both descriptions. In one of them, the Yang-Mills theory has gauge group $\mathrm{U}(N)$ and is defined on a time coordinate multiplied by a two-torus with circumferences $1/R_1, 1/R_2$ (instead of 2π employed in the previous section) where R_1, R_2 are the radii of the spacetime circles in Planck units[1] and the fermions are nonlocal degrees of freedom (arrows) pointing from the point (σ_1, σ_2) to the point $(\sigma_1, \sigma_2 + 1/2R_2)$. We will call this picture "nonlocal".

We will also sometimes use a "local" picture where the coordinate σ_2 is wrapped twice and its circumference is only $1/2R_2$. In the local picture, the gauge group is $\mathrm{U}(N) \times \mathrm{U}(N)$ and these two factors exchange when we go around the σ_2 circle so that the "effective" period is still equal to $1/R_2$:

$$A^{\alpha}_{ij}\left(\sigma_0, \sigma_1, \sigma_2 + \frac{1}{2R_2}\right) = A^{1-\alpha}_{ij}(\sigma_0, \sigma_1, \sigma_2)\,. \tag{2.6}$$

Here $\alpha = 0, 1$ is an index distinguishing the two factors in $\mathrm{U}(N) \times \mathrm{U}(N)$. We suppresed the worldvolume vector index $\mu = 0, 1, 2$. Indices i, j run from 1 to N; here i spans $\mathbf{N}$ and j belongs to $\bar{\mathbf{N}}$. Similar boundary conditions are imposed on the scalars X which also transform in the adjoint of $\mathrm{U}(N) \times \mathrm{U}(N)$. Both satisfy the usual hermiticity conditions. Fermions θ (whose spacetime transformation rules are the same as in the supersymmetric theory) transform in $(\mathbf{N}, \bar{\mathbf{N}})$. Writing them as θ_{ij}, the index i belongs to $\mathbf{N}$ of the first $\mathrm{U}(N)$ and the index j belongs to $\bar{\mathbf{N}}$ of the second $\mathrm{U}(N)$. In the same way, in $(\theta^\dagger)_{ij} = (\theta_{ji})^\dagger$ the first index i belongs to $\mathbf{N}$ of the second $\mathrm{U}(N)$ and the second index j belongs to $\bar{\mathbf{N}}$ of the first $\mathrm{U}(N)$ so that $\mathrm{Tr}\,\theta^\dagger\theta = \theta^\dagger_{ij}\theta_{ji}$ is invariant. The boundary condition for θs reads

$$\theta_{ij}\left(\sigma_0, \sigma_1, \sigma_2 + \frac{1}{2R_2}\right) = (\theta^\dagger)_{ij}(\sigma_0, \sigma_1, \sigma_2)\,. \tag{2.7}$$

Of course, θ matrices are complex, they do not obey a hermiticity condition. The lagrangian is $(i = 1, \ldots, 7)$

$$\begin{aligned}\mathcal{L} = \sum_{\alpha=0,1} \mathrm{Tr}\left[-\frac{1}{4}F^{\mu\nu}_{(\alpha)}F_{(\alpha),\mu\nu} - \frac{1}{2}D_\mu X^i_{(\alpha)} D^\mu X^i_{(\alpha)} + \frac{1}{4}[X^i_{(\alpha)}, X^j_{(\alpha)}]^2\right] + \\ + \mathrm{Tr}\left[i\theta^\dagger\gamma^i X^i_{(\alpha=0)}\theta + i\theta\gamma^i X^i_{(\alpha=1)}\theta^\dagger + \theta^\dagger\gamma_\mu\partial^\mu\theta + i\theta^\dagger\gamma_\mu A^\mu_{(\alpha=0)}\theta + i\theta\gamma_\mu A^\mu_{(\alpha=1)}\theta^\dagger\right].\end{aligned} \tag{2.8}$$

The trace always runs over $N \times N$ matrices. We have put the dimensionful quantity g_{YM} equal to one. The action is simply

$$\mathcal{A} = \int d\sigma^0 \int_0^{1/R_1} d\sigma^1 \int_0^{1/(2R_2)} d\sigma^2 \mathcal{L}(\sigma^0, \sigma^1, \sigma^2)\,. \tag{2.9}$$

[1] To simplify the presentation, we choose the convention for the numerical constants in various dimensionful quantities to agree with these statements.

For the purposes of the calculations of Feynman diagrams it is also useful to write the action in the nonlocal (nl) formulation of the theory. In this formulation, the period of σ^2 is doubled and equal to $1/R_2$. The fields can be identified as follows (the dependences on σ^0, σ^1 and indices μ, i are suppressed):

$$X_{(\alpha)}(\sigma^2) = X_{nl}\left(\sigma^2 + \frac{\alpha}{(2R_2)}\right), \qquad A_{(\alpha)}(\sigma^2) = A_{nl}\left(\sigma^2 + \frac{\alpha}{(2R_2)}\right), \qquad \alpha = 0, 1, \tag{2.10}$$
$$\theta(\sigma^2) = \theta_{nl}(\sigma^2)\,, \qquad \theta^\dagger(\sigma^2) = \theta^\dagger_{nl}(\sigma^2) = \theta_{nl}\left(\sigma^2 + \frac{1}{(2R_2)}\right), \quad 0 \le \sigma^2 \le \frac{1}{2R_2}\,.$$

All the equalities are $N \times N$ matrix equalities. In this nonlocal language the action can be written as

$$\mathcal{A} = \int d\sigma^0 \int_0^{1/R_1} d\sigma^1 \int_0^{1/R_2} d\sigma^2 \mathcal{L}_{nl}(\sigma^0, \sigma^1, \sigma^2)\,, \tag{2.11}$$

where (the subscript "nl" of all fields is suppressed)

$$\begin{aligned}\mathcal{L}_{nl} = &\operatorname{Tr}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}D_\mu X^i D^\mu X^i + \frac{1}{4}[X^i, X^j]^2\right] + \\ &+ \operatorname{Tr}\left[\theta^\dagger \gamma_\mu \partial^\mu \theta + i\theta^\dagger(\gamma^i X^i(\sigma^2) + \gamma_\mu A^\mu(\sigma^2))\theta\right] + \\ &+ \operatorname{Tr}\left[i\theta\left(\gamma^i X^i(\sigma^2 + \frac{1}{2R_2}) + \gamma_\mu A^\mu(\sigma^2 + \frac{1}{2R_2})\right)\theta^\dagger\right].\end{aligned} \tag{2.12}$$

We denoted the σ^2 dependence in which one of the fermionic terms is nonlocal.

3. Alternative derivation and connection with type-0 theories

It has long been known [4, 5] that the type-$0_{A,B}$ string theories in ten dimensions can be viewed as infinite temperature limits of type-$\mathrm{II}_{B,A}$ theories. Rotating the Euclidean time to a spacelike direction, this means that the zero radius limits of Rohm compactifications are the type-0 theories. It is less well known (but, we believe, known to many experts) that the finite radius Rohm compactifications are compactifications of the type-0 theories on dual circles with a certain orbifold projection. Indeed, both type-0 theories have two types of Ramond-Ramond fields which are related by a discrete symmetry. This doubled number is a consequence of having one GSO-projection only (the diagonal one). More precisely, the operator $\mathcal{R} = (-1)^{F_R}$ which counts the right-moving fermionic excitations has eigenvalues $(+1)$ for half of the RR-fields and (-1) for the other half. Therefore $(-1)^{F_R}$ is a generator of a $\mathbb{Z}_2$ symmetry that exchanges the RR-fields in a basis rotated by 45 degrees, i.e. $\mathrm{RR}_{+1} + \mathrm{RR}_{-1}$ with $\mathrm{RR}_{+1} - \mathrm{RR}_{-1}$ where $\mathrm{RR}_{\pm 1}$ denotes the fields with $(-1)^{F_R} = \pm 1$.

Twisted compactification of type-0A or -0B with monodromy $\mathcal{R}$ i.e. the orbifold of type-0 string theory on a circle of circumference $2L$ by the symmetry $\mathcal{R}\,\exp(iLp)$

gives a string model T-dual to the Rohm compactified type-IIB or type-IIA string, respectively. We will refer to the twisted circle as the Scherk-Schwarz circle when describing it from the type-II point of view and as the $\mathcal{R}$ circle from the type-0 point of view.

This T-duality is not hard to understand at the level of the string spectrum. Because of the GSO projection, Scherk-Schwarz compactified type-II theory contains bosonic states of integer momenta and fermionic states of half-integer momenta (in appropriate units). The dual type-0 string theory initially had bosonic excitations only (in NS-NS and RR sectors). But because of the extra orbifold by $\mathcal{R}\ \exp(iLp)$, we obtain also fermions in NS-R and R-NS twisted sectors with a half-integer winding. This agrees with the assumption of T-duality. Apart from T-duality between type-IIA/IIB on a Scherk-Schwarz circle and type-0B/0A on an $\mathcal{R}$ circle which we just mentioned, we should be aware of the T-duality between type-0A and type-0B string theory on a usual circle.

Now consider the DLCQ of M-theory compactified on a Scherk-Schwarz circle. Using the logic of [2], this is a zero coupling limit of type-IIA string theory compactified on a Scherk-Schwarz circle of Planck size, in the presence of N D0-branes. Using the T-duality adumbrated in the previous paragraphs, this is weakly coupled type-0B string theory on an $\mathcal{R}$ circle in the presence of N D-strings of the first kind; since the $\mathcal{R}$ monodromy exchanges two types of D-strings, there must be an equal number of D-strings of the other type. In other words, the D-strings are compactified on a circle dual to the M-theory Scherk-Schwarz circle, with $\mathcal{R}$ twisted boundary conditions.

Now we can use the description of D-branes in type-0 theories discovered by Bergman and Gaberdiel [13]. As we have said, there are two types of D-strings in type-0B theory, each of which has a bosonic $1+1$ dimensional gauge theory on its world volume: open strings stretched between two like D-strings contain bosonic states only. These two types are related (exchanged) by the $\mathcal{R}$ symmetry. In the presence of closely spaced D-strings of both types, there are additional fermionic degrees of freedom which transform in the $(N,\bar{M})[\oplus(\bar{N},M)]$ of the $\mathrm{U}(N)\times\mathrm{U}(M)$ gauge group: open strings stretched between two unlike D-strings contain fermions only. These fermions are spacetime spinors. In the corresponding Seiberg limit, all the closed string states (including tachyons) are decoupled and in the corresponding DKPS energy scale only the massless open string states survive.

The result is a $1+1$ dimensional $\mathrm{U}(N)\times\mathrm{U}(N)$ gauge theory with fermions in the bifundamental and a boundary condition that exchanges the two $\mathrm{U}(N)$ groups as we go around the circle, a result of the $\mathcal{R}$ monodromy. This is the same gauge theory we arrived at by the orbifolding procedure of the previous section.

It is now easy to compactify an extra dimension on an ordinary circle and obtain the $2+1$ dimensional gauge theory of the previous section as the matrix description of Rohm compactification. The double T-duality in Seiberg's derivation can be done in two possible orders, giving always the same result. In the next section, we will see that at least formally we can rederive the various string theories as matrix string limits of the gauge theory. In particular, this will provide a derivation of the Bergman-Gaberdiel duality relation between Scherk-Schwarz compactification of M-theory, and type-0A strings.

4. The matrix string limits

4.1 Rohm compactified type-IIA strings

In the limit where the spacetime radius R_2 goes to infinity, the radius of the worldvolume torus $1/R_2$ goes to zero so that we also have $1/R_1 \gg 1/R_2$. Therefore the fields become effectively independent of σ^2 up to a gauge transformation. Furthermore, because of the boundary conditions exchanging the two $\mathrm{U}(N)$'s, the expectation values of scalars in both $\mathrm{U}(N)$'s must be equal to each other (up to a gauge transformation). This can be also seen in the nonlocal formulation: in the limit $R_2 \to \infty$ the fields must be constant (up to a gauge transformation) on the long circle of circumference $1/R_2$.

Thus in this limit we can classify all the fields according to how they transform under the σ^2 independent gauge symmetry $\mathrm{U}(N)$. In the nonlocal language this $\mathrm{U}(N)$ is just a "global" (but σ^1 dependent) symmetry. In the $\mathrm{U}(N) \times \mathrm{U}(N)$ language this is the diagonal symmetry $\mathrm{U}(N)$. In both cases, we find that not only bosons but also fermions (transforming originally in $(\mathbf{N}, \bar{\mathbf{N}})$) transform in the adjoint (the same as $\mathbf{N} \otimes \bar{\mathbf{N}}$) of this $\mathrm{U}(N)$. There is only one set of fields: the σ^2 independence causes the bosons in both $\mathrm{U}(N)$'s to be equal and the complex matrices θ to be hermitean.

In the matrix string limit we expect to get a matrix description of the Scherk-Schwarz compactification of type-IIA strings on a long circle. The appearance of the matrix strings (at a naive level) can be explained as usual: most things work much like in the supersymmetric matrix string theory [15, 16, 17].

In the nonlocal formulation, $\mathrm{U}(N)$ gauge group is broken completely down to a semidirect product of $\mathrm{U}(1)^N$ and the Weyl group, S_N, of $\mathrm{U}(N)$. Therefore the classical configurations around which we expand are diagonalizable $N \times N$ matrices where the basis in which they can be diagonalized can undergo a permutation $p \in S_N$ for $\sigma^1 \to \sigma^1 + 1/R_1$:

$$X_i(\sigma^1) = U(\sigma^1)\, \mathrm{diag}(x_i^1, x_i^2, \dots x_i^n) U^{-1}, \qquad \mathrm{U}\left(\sigma^1 + \frac{1}{R_1}\right) = U(\sigma^1) p\,. \tag{4.1}$$

This is the mechanism of matrix strings [15, 16, 17]. Every permutation p can be decomposed into a product of cycles and each cycle of length k then effectively describe a "long string" with the longitudinal momentum equal to $p^+ = k/R^-$. For instance, a single cyclic permutation of k entries (written as a $k \times k$ matrix p)

describes a single string:

$$p = \begin{pmatrix} \circ & 1 & \circ & \dots & \circ \\ \circ & \circ & 1 & \dots & \circ \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 1 & \circ & \circ & \dots & \circ \end{pmatrix}. \tag{4.2}$$

The definition (4.1) of X_i creates effectively a string of length k (relatively to the circumference $1/R_1$). We can write the eigenvalues as

$$x_i^m(\sigma^1) = x_i^{\text{long}}\left(\sigma^1 + \frac{(m-1)}{R_1}\right), \qquad m = 1, 2, \dots, k \tag{4.3}$$

where x_i^{long} has period k/R_1. Assuming the k/R_1 periodicity of x_i^{long} we can show $1/R_1$ periodicity of the matrix (4.1) with p defined in (4.2).

The matrix origin of the level-matching conditions was first explained in [17]: the residual symmetry $\mathbb{Z}_k$ rotating the "long" string is a gauge symmetry and because the states must be invariant under the gauge transformations, we find out that $L_0 - \tilde{L}_0$ must be a multiple of k (the length of the string) because the generator of $\mathbb{Z}_k$ can be written as $\exp(i(L_0 - \tilde{L}_0)/k)$. In the large-$N$ limit such states are very heavy unless $L_0 = \tilde{L}_0$ and we reproduce the usual level-matching conditions. In this limit the discrete group $\mathbb{Z}_k$ approximates the continuous group quite well.

4.2 Dependence on the fluxes

In the conformal field theory of the Rohm compactification, sectors with odd or even winding numbers should have antiperiodic or periodic spinors θ, respectively. We want to find the analog of this statement in the Matrix formulation.

Let us put w units of the magnetic flux in the nonlocal representation of our theory. The corresponding potential can be taken to be

$$A_{\mu=1} = 2\pi R_1 R_2 \sigma_2 \frac{w}{N}, \qquad A_{\mu=2} = 0 \tag{4.4}$$

Recall that the periods of σ_1, σ_2 are $1/R_1, 1/R_2$. In the local $\mathrm{U}(N) \times \mathrm{U}(N)$ formulation, the fields in the region $0 \le \sigma_2 \le 1/2R_2$ from (4.4) define the block of the first $\mathrm{U}(N)$ and the region $1/2R_2 \le \sigma_2 \le 1/R_2$ defines the second $\mathrm{U}(N)$. Note that for $\sigma_2 \to \sigma_2 + 1/R_2$, $\mathrm{Tr}\, A_{\mu=1}$ changes by $2\pi R_1 w$ which agrees with the circumference of X^1.

Now if we substitute the background (4.4) into (2.8) we see that the contributions of the form $\theta A \theta$ from the last two terms give us a contribution coming from the difference $\sigma_2 \to \sigma_2 + 1/2R_2$ which is equal to

$$i\, \mathrm{Tr}[\theta^\dagger \gamma_1 \theta] \frac{\pi w R_1}{N}. \tag{4.5}$$

Such a term without derivatives would make the dynamics nonstandard. However it is easy to get rid of it by a simple redefinition (we suppress σ_0, σ_2 dependence)

$$\theta(\sigma_1) \to \theta(\sigma_1) \exp\left(\frac{i\sigma_1 R_1 \pi w}{N}\right). \tag{4.6}$$

Note that under $\sigma_1 \to \sigma_1 + N/R_1$ which corresponds to a loop around a matrix string of length N, θ changes by a factor of $(-1)^w$. This confirms our expectations: in the sectors with an odd magnetic flux (= winding number) the fermions θ are antiperiodic.

We might also wonder about the electric flux (= compact momentum p_2) in the direction of σ_2. As we have explained in the beginning, this flux should be allowed to take 1/2 of the original quantum so that the sectors with half-integer electric flux contain just fermions and the usual sectors with integer electric flux contain bosons only, the other being projected out by the GSO conditions in both cases. This behaviour should be guaranteed "by definition": the operator $\exp(2\pi R_2 \hat{p}_2)(-1)^F$ is identified with a gauge transformation (namely $\exp(2\pi R_2 \sigma_2) \otimes \mathbb{1}_{N\times N}$).

However it might seem a little strange that the together with the θ excitations one must also change the electric flux; it might be useful to see the origin of the sectors of various electric flux "microscopically". We propose the following way to think about this issue. The θ excitations in a compact space carry charge $\pm$ with respect to groups U(1) at the opposite points of σ_2. The total charge vanishes therefore we do not have an obstruction to excite θ. However the charge does not vanish locally, therefore we should accompany the excitation by an electric flux tube running between σ_2 and $\sigma_2 + 1/2R_2$ in a chosen direction (it is useful to think about it as a "branch-cut") and the total electric flux induced by this excitation equals one half of the quantum in the supersymmetric (untwisted) theory.

As a consequence of these observations, we see that our model contains the string field theory Hilbert space of the Rohm compactification in the large-N limit. At a very formal level, the dynamics of the model in an appropriate limit of small radii and large Yang-Mills coupling, appears to reduce to that of free Rohm strings. However, this is not necessarily a correct conclusion. The analysis of the moduli space lagrangian is done at the classical level, but the apparent free string limit corresponds to a strongly coupled YM theory. In [16] it was emphasized that the derivation of Matrix string theory depends crucially on the nonrenormalization theorem for the moduli space lagrangian. We do not have such a theorem here and cannot truly derive the free Rohm string theory from out model. This is only the first of many difficulties.

A further important point is that the U(1) gauge theory (or U(1) $\times$ U(1) in the local formalism) leads to a free theory which is identical to the conformal field theory in the matrix string limit $1/R_1 \gg 1/R_2$. In particular we can see that the ground state in the sectors with an odd magnetic flux has negative light cone energy, and would have to be interpreted as a tachyon in a relativistic theory. Even if we assumed the clustering property to be correct (in the next section we show that this property is likely to be broken at the two-loop level), this tachyon would lead to inconsistency in the large-N limit: it would be energetically favoured for a configuration in the $\mathrm{U}(N)$ theory to emit the $N=1$ tachyonic string — and compensate the magnetic flux by the opposite value of the flux in the remaining $\mathrm{U}(N-1)$ theory. The energy of tachyon is of order $-N^0$ which is negative and N times bigger than the scale of energies we would hope to study in the large-N limit (only states with energies of order $1/N$ admit a relativistic interpretation in the large-N limit).

To make this more clear: in order to establish the existence of a relativistic large-N limit we would have to find states with dispersion relation $\frac{\mathbf{p}^2+m^2}{N}$ in the model, as well as multiparticle states corresponding to separated particles which scatter in a manner consistent with relativity. The observation of the previous paragraph shows that such states would generally be unstable to emission of tachyons carrying the smallest unit of longitudinal momentum.[2] The only way to prevent this disaster is to lift the moduli space. However, once we imagine that the moduli space is lifted it is unlikely that multiparticle states of any kind exist and the model loses all possible spacetime interpretation. We will investigate the cluster property of our model below. However, we first want to investigate the type-0 string limits of our model. As above, we will work in a purely classical manner and ignore the fact that the moduli space is lifted by quantum corrections.

4.3 Type-0 matrix strings

The Rohm compactified IIA string is the formal limit of our $2+1$ dimensional gauge theory when the untwisted circle of the Yang-Mills torus is much larger than the scale defined by the gauge coupling, while the twisted circle is of order this scale or smaller. We will now consider three other limits. The relation between the Yang-Mills parameters and the M-theory parameters is

$$g_{YM}^2 = \frac{R}{L_1\,L_2} \qquad \Sigma_i = \frac{l_{\mathrm{planck}}^3}{R\,L_i}\,, \tag{4.7}$$

where $\Sigma_{1,2}$ is the untwisted (twisted) YM radius, $L_{1,2}$ are the corresponding M-theory radii, and R is the lightlike compactification length. In the type-0 string limit, we want to take $L_2 \to 0$, with Σ_2 fixed (it is the string length squared divided by R) and L_1 of order the string length). The latter restriction means that $g_{YM}^2\Sigma_1$ is fixed. The limit is thus a $1+1$ dimensional gauge theory on a fixed length twisted circle, with gauge coupling going to infinity.

[2] Note that in SUSY Matrix Theory the excitations along directions where the gauge group is completely broken down to $\mathrm{U}(1)$ factors have *higher* energy than the states with large longitudinal momentum.

Restricting ourselves to classical considerations, we are led to the classical moduli space of this gauge theory. The bosonic sector of the moduli space consists of two sets of independent $N \times N$ diagonalizable matrices. However, in order to obtain configurations which obey the twisted boundary conditions and have energy of order $1/N$, one must consider only topological sectors in which the matrices in the two gauge groups are identical. Note however, that since the bosonic variables are in the adjoint representation, they are not affected by gauge transformations which are in the U(1) subgroup. This additional freedom becomes important when we consider the fermionic variables. The boundary conditions on these allow one other kind of configuration with energy of order $1/N$: considering the fermions as $N \times N$ matrices, we can allow configurations in which the diagonal matrix elements come back to minus themselves (corresponding to the gauge transformation $\pm(1, -1)$ in $\mathrm{U}(N) \times \mathrm{U}(N)$) after a cycle with length of order N. The resulting low energy degrees of freedom are fermion fields on the "long string" with either periodic or antiperiodic boundary conditions. The gauge fields are vector like so in terms of left and right moving fields we get only the PP and AA combinations of boundary conditions. The $O(8)$ chirality of the fermions is correlated to the world sheet chirality as in IIA matrix string theory. One also obtains a GSO projection on these fermionic degrees of freedom by imposing the gauge projection corresponding to the $(1, -1)$ transformation. The resulting model is thus seen to be the type-0A string theory, written in light cone Green-Schwarz variables. Remembering that the $1+1$ twisted gauge theory was the matrix description of Scherk-Schwarz compactification of M-theory, we recognize that we have derived the conjecture of Bergmann and Gaberdiel.

To obtain the 0B matrix string limit and the T-duality (on an untwisted circle) between the two type-0 theories, we simply follow the results of one of the present authors and Seiberg [16] and first take the strongly coupled Yang-Mills limit by going to the classical moduli space and performing a $2+1$ dimensional duality transformation. This corresponds to both directions of the Yang-Mills torus being much larger than the Yang-Mills scale. We then do a dimensional reduction to a $1+1$ dimensional theory to describe the 0B and IIB string limits. In the former, the twisted circle is taken much larger than the untwisted one, while their relative sizes are reversed in the latter limit. After the duality transformation and dimensional reduction the manipulations are identical to those reported above.

A serious gap in the argument is the absence of $2+1$ superconformal invariance. In [16] this was the crucial fact that enabled one to show that the interacting type-IIB theory was Lorentz invariant. Here that argument fails. We view this as an indication that the spacetime picture derived from free type-0 string theory is misleading. We will discuss this further below. Indeed, in the next subsection we show that the cluster property which is at the heart of the derivation of spacetime from Matrix Theory fails to hold in our model.

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4.4 Breakdown of the cluster property

The easiest way to derive the Feynman rules is to use the nonlocal formulation (2.12). It looks similar to a local lagrangian except that the gauge field in the last term (i.e. the whole third line) is taken from $\sigma^2 + 1/2R^2$. In the Feynman diagrams the propagators have (worldvolume) momenta in the lattice corresponding to the compactification, i.e. P_1, P_2 are multiples of $2\pi R_1$ or $2\pi R_2$ respectively. The last term in (2.12) gives us a vertex with two fermions and one gauge boson and the corresponding Feynman vertex contains a factor $(-1)^{P^2/2\pi R_2}$.

In order to determine the cluster properties of our theory, we must calculate the effective action along the flat directions in the classical moduli space of the gauge theory. We will concentrate on a single direction in which (in the nonlocal formulation) the gauge group is broken to $\mathrm{U}(N_1) \times \mathrm{U}(N_2)$. That is, we calculate two body forces, rather than general k body interactions. There is a subtlety in this calculation which has to do with our lack of knowledge of the spectrum of this nonsupersymmetric theory.

In general, one may question the validity of the Born-Oppenheimer approximation for the flat directions because the individual nonabelian gauge groups appear to give rise to infrared divergences in perturbation theory. In the SUSY version of Matrix Theory this problem is resolved by the (folk) theorem that the general $\mathrm{U}(N)$ theory (compactified on a torus) has threshold bound states. These correspond to wave functions normalizable along the flat directions and should cut off the infrared divergences. In our SUSY violating model, we do not know the relevant theorems.

The most conservative way to interpret our calculation is to take $N_1 = N_2 = 1$ in the $\mathrm{U}(2)$ version of the model. If one finds an attractive two body force then it is reasonable to imagine that in fact the general $\mathrm{U}(N)$ theory has a normalizable ground state, thus justifying the Born-Oppenheimer approximation in the general case.

So let us proceed to calculate the potential in the $\mathrm{U}(2)$ case, and let R be the field which represents the separation between two excitations of the $\mathrm{U}(1)$ model. From the point of view of $2+1$ dimensional field theory, R is a scalar field, with mass dimension $1/2$. It is related to the distance measured in M-theory by powers of the eleven dimensional Planck scale. At large R, the charged fields of the $\mathrm{U}(2)$ model are very heavy. To integrate them out we must understand the UV physics of the model. The formulation in terms of a $\mathrm{U}(2) \times \mathrm{U}(2)$ theory with peculiar boundary conditions shows us that the UV divergences are of the same degree as those of the SUSY model, though some of the SUSY cancellations do not occur, as we will see below. Ultraviolet physics is thus dominated by the fixed point at vanishing Yang-Mills coupling and we can compute the large R expansion of the effective action by perturbation theory.

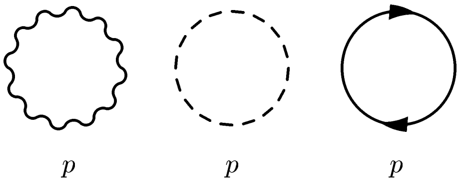

Figure 1: One-loop diagrams.

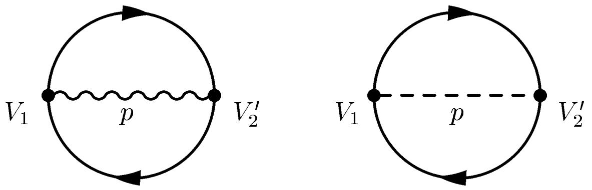

Figure 2: Non-vanishing two-loop contributions to the effective potential. Here $V_2' \equiv (\epsilon(p) - 1)V_2$.

The one loop contribution, figure 1, to the effective potential vanishes because it is identical to that in the SUSY model. The only difference between the models in the nonlocal formulation is the peculiar vertex described above. The leading contribution comes from two loops and is of order g_{YM}^2 (the squared coupling has dimensions of mass). It comes only from the diagrams containing fermion lines shown in figure 2. These diagrams should be evaluated in the nonlocal model and then their value in the SUSY model should be subtracted. The rest of the two loop diagrams in the model are the same as the SUSY case and they cancel (for time independent R) against the SUSY values of the diagrams shown. Taking R very large in the diagrams is, by dimensional analysis, equivalent to taking the volume large, and the potential is extensive in the volume in the large volume limit. The massive particles in the loops have masses of order $g_{YM}R$ and this quantity is kept fixed in the loop expansion.

Our lagrangian has two gauge boson fermion vertices $V_1 + \epsilon(p)V_2$. p is the momentum. In the SUSY theory, these are the two terms in the commutator. In our lagrangian, the first is identical to that in the SUSY theory while the second differs from it by the sign $\epsilon(p)$, which is negative for odd values of the loop momentum around the twisted circle. Schematically then, the nonvanishing two loop contribution has the form

$$\langle (V_1 + \epsilon(p)V_2)^2 \rangle - \langle (V_1 + V_2)^2 \rangle \tag{4.8}$$

This can be rewritten as

$$2\langle (\epsilon(p) - 1)V_1 V_2 \rangle \,. \tag{4.9}$$

The resulting loop integral is quadratically ultraviolet divergent. The leading divergence is independent of R, but there are subleading terms of order $g_{YM}^2 \Lambda \, |g_{YM}R|$ and $g_{YM}^2 \ln(\Lambda)(g_Y MR)^2$. Corrections higher order in the Yang-Mills coupling, as well as those coming from finite volume of the Yang-Mills torus, are subleading both in R and Λ. Thus, the leading order contribution to the potential is either confining, or gives a disastrous runaway to large R. Which of these is the correct behavior is determined by our choice of subtractions. It would seem absurd to choose the renormalized coefficient of R^2 to be negative, and obtain a hamiltonian unbounded from below. If that is the case, then a confining potential prevents excitations from separating from each other in the would-be transverse spacetime. In other words, the theory does not have a spacetime interpretation at all, let alone a relativistically invariant one.

We also see that the fear expressed in the previous chapter that all excitations will decay into tachyons of minimal longitudinal momentum was ill founded. Instead it would appear that the entire system will form a single clump in transverse space. The U(1) part of the theory decouples, so we can give this clump transverse momentum and obtain an energy spectrum

$$P^- \sim \frac{R\mathbf{P}^2}{N} + \Delta\,. \tag{4.10}$$

where Δ is the ground state energy of our nonlocal SU(N) Yang-Mills theory.

If Δ were to turn out positive and of order $1/N$, this dispersion relation would look like that of a massive relativistic particle. We might be tempted to say that the system looked like a single black hole propagating in an asymptotically flat spacetime. The stability of the black hole would be explained if its mass were within the Planck regime. This interpretation does not appear to be consistent, for semiclassical analysis of such a system indicates that it has excitations corresponding to asymptotic gravitons propagating in the black hole background. Our result about the lifting of the moduli space precludes the existence of such excitations.

Since the vacuum energy is divergent, the positivity of Δ is a matter of choice. However, large-N analysis suggests that it scales like a positive power of N, so we have another reason that the black hole interpretation does not seem viable.

5. Conclusions

What are we to make of all these disasters? We believe that our work is solid evidence for the absence of a Lorentz invariant vacuum of M-theory based on the Rohm compactification. The Rohm strings are certainly degrees of freedom of our matrix model, even if we cannot derive the (apparently meaningless because of the tachyon and unbounded effective potential) string perturbation expansion from it (as a consequence of the absence of a nonrenormalization theorem).

However, naive physical intuition based on the string perturbation series, suggest that if there is a stable solution corresponding to the Rohm model, it is not Lorentz invariant. While we cannot trust the perturbative calculations in detail, they at least imply that the vacuum energy of the system is negative at its (hypothetical) stable minimum. (We remind the reader that even at large radius, before the tachyon appears, the potential calculated by Rohm is negative and the system wants to flow to smaller radius). Perhaps there is a nonsupersymmetric anti-de Sitter solution of M-theory to which the Rohm model "flows". There are many problems with such an interpretation, since it involves changing asymptotic boundary conditions in a generally covariant theory. Normally one would imagine that M-theory with two different sets of asymptotic boundary conditions breaks up into two different quantum mechanical systems which simply do not talk to each other.

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The finite energy states with one set of boundary conditions simply have no overlap with the finite energy states of another (the definition of energy is completely different).

As an aside we note that an extremely interesting question arises for systems (unlike the Rohm compactification) which have a *metastable* Minkowski vacuum. In the semiclassical approximation [8] one can sometimes find instantons which represent tunneling of a Minkowski vacuum into a "bubble of anti-de Sitter space". Does this idea make any sense in a fully quantum mechanical theory, particularly if one believes in the holographic principle? Coleman and De Luccia argue that the system inside the AdS bubble is unstable to recollapse and interpret this as a disaster of cosmic proportions: Minkowski space fills up with bubbles, expanding at the speed of light, the interior of each of which becomes singular in finite proper time. It is hard to imagine how such a scenario could be described in a holographic framework.[3]

At any rate, it is clear that the fate of the Rohm compactification depends crucially on a change in vacuum expectation values. In this sense one might argue that our attempt to study it in light cone frame was "doomed from the start". It is a notorious defect of the light cone approach that finding the correct vacuum is extremely difficult. It involves understanding and cancelling the large-N divergences of the limiting DLCQ, by changing parameters in the light cone hamiltonian. If the correct vacuum is a finite distance away in field space from the naive vacuum from which one constructs the original DLCQ hamiltonian, this may simply mean that the true hamiltonian has little resemblance to the one from which one starts. If our physical arguments above are a good guide, the problem may be even more severe. The correct vacuum may not even have a light cone frame hamiltonian formulation.

We confess to having jumped in to the technical details of our construction before thinking through the physical arguments above. Nonetheless, we feel that our failure is a useful reminder that gravitational physics is very different from quantum field theory, and an indication of the extreme delicacy of SUSY breaking in M-theory.

Acknowledgments

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[3]This should not be taken simply as an indication that a holographic description of asymptotically flat spacetimes is somehow sick. The Coleman-De Luccia instanton also exists in an asymptotically AdS framework, with two negative energy vacua. If the higher energy state has very small vacuum energy the semiclassical analysis is practically unchanged. So, if the Coleman-De Luccia phenomenon really exists in M-theory we should be able to find a framework for studying it within AdS/CFT.

References

[1] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[2] N. Seiberg, *Why is the matrix model correct?*, *Phys. Rev. Lett.* **79** (1997) 3577 [hep-th/9710009].

[3] I.W. Taylor, *D-brane field theory on compact spaces*, *Phys. Lett.* **B 394** (1997) 283 [hep-th/9611042].

[4] N. Seiberg and E. Witten, *Spin structures in string theory*, *Nucl. Phys.* **B 276** (1986) 272.

[5] L.J. Dixon and J.A. Harvey, *String theories in ten-dimensions without space-time supersymmetry*, *Nucl. Phys.* **B 274** (1986) 93.

[6] J. Scherk and J.H. Schwarz, *Spontaneous breaking of supersymmetry through dimensional reduction*, *Phys. Lett.* **B 82** (1979) 60.

[7] R. Rohm, *Spontaneous supersymmetry breaking in supersymmetric string theories*, *Nucl. Phys.* **B 237** (1984) 553.

[8] S. Coleman and F.D. Luccia, *Gravitational effects on and of vacuum decay*, *Phys. Rev.* **D 21** (1980) 3305.

[9] L. Motl, *Quaternions and M(atrix) theory in spaces with boundaries*, hep-th/9612198.

[10] M.R. Douglas and G. Moore, *D-branes, quivers and ALE instantons*, hep-th/9603167.

[11] T. Banks and L. Motl, *Heterotic strings from matrices*, *J. High Energy Phys.* **12** (1997) 004 [hep-th/9703218];
T. Banks, W. Fischler and L. Motl, *Dualities versus singularities*, *J. High Energy Phys.* **01** (1999) 019 [hep-th/9811194].

[12] E. Witten, *String theory dynamics in various dimensions*, *Nucl. Phys.* **B 443** (1995) 85 [hep-th/9503124].

[13] O. Bergman and M.R. Gaberdiel, *Dualities of type-0 strings*, *J. High Energy Phys.* **07** (1999) 022 [hep-th/9906055].

[14] Y. Imamura, *Branes in type-0/type II duality*, *Prog. Theor. Phys.* **102** (1999) 859 [hep-th/9906090].

[15] L. Motl, *Proposals on nonperturbative superstring interactions*, hep-th/9701025.

[16] V.P. Nair and S. Randjbar-Daemi, *On brane solutions in M(atrix) theory*, *Nucl. Phys.* **B 533** (1998) 333 [hep-th/9802187].

[17] R. Dijkgraaf, E. Verlinde and H. Verlinde, *Matrix string theory*, *Nucl. Phys.* **B 500** (1997) 43 [hep-th/9703030].

[18] J.P. Penalba, *A matrix model for type-0 strings*, *J. High Energy Phys.* **08** (1999) 005 [hep-th/9907083].

[19] R. Blumenhagen, C. Kounnas and D. Lust, *Continuous gauge and supersymmetry breaking for open strings on D-branes*, *J. High Energy Phys.* **01** (2000) 036 [hep-th/9910094].